Predicting maximum branch diameter from crown dimensions, stand characteristics and tree species

by Arthur Groot¹ and Robert Schneider²

ABSTRACT
Forest resource inventories must include wood quality information to support the optimum use of wood fibre. The objective of this study was to develop models relating maximum live branch diameter (MBD), which affects lumber value, to tree and stand characteristics that can be measured through current and emerging remote sensing technologies. Using non-linear mixed effects models for six Canadian conifer species, as well as for three broad-leaved species, MBD was related to crown radius, tree height, crown length, stand basal area, and basal area of trees larger than the subject tree. Models that included only individual tree characteristics (crown radius, tree height, and crown length) did not perform as well as models that additionally included stand characteristics (stand basal area and basal area of larger trees). Models that took into account tree species performed better than models that did not; in particular, broadleaved species had much thicker branches than conifers. The best model did not show bias with respect to independent variables and had root mean square error of 0.32 cm. For the best model, prediction error was not related to silvicultural treatment. These model characteristics strongly support the potential to successfully predict MBD from remotely sensed data.

Key words: branch diameter, model, forest inventory

RÉSUMÉ
Les inventaires des ressources forestières doivent inclure des informations sur la qualité de la matière ligneuse afin de pouvoir utiliser de façon optimale cette dernière. L’objectif de cette étude visait à élaborer des modèles reliant le diamètre maximal des branches vivantes (DMB), qui a une influence sur la valeur du bois de sciage, aux caractéristiques des arbres et du peuplement qui peuvent être mesurées au moyen des technologies actuelles et émergentes de télédétection. En utilisant des modèles non linéaires à effets variables pour six espèces de conifères du Canada, ainsi que pour trois espèces de feuillus, le DMB a été relié au diamètre de la cime, à la hauteur de l’arbre, à la longueur de la cime, à la surface terrière du peuplement et à la surface terrière des arbres plus gros que l’arbre étudié. Les modèles qui comprenaient également les caractéristiques individuelles des arbres (diamètre de la cime, hauteur de l’arbre et longueur de la cime) n’ont pas performé aussi bien que les modèles qui comprenaient, en plus, les caractéristiques du peuplement (surface terrière du peuplement et surface terrière des arbres plus gros). Les modèles qui ont tenu compte des espèces d’arbres ont mieux performé que les modèles qui n’en tenaient pas compte; notamment, les espèces feuillues avaient des branches beaucoup plus grosses en diamètre que les conifères. Le meilleur modèle n’a pas démontré de biais en fonction des variables indépendantes et affichait une erreur quadratique moyenne de 0,32 cm. Dans le cas du meilleur modèle, l’erreur de prédiction n’était pas reliée au traitement sylvicole. Ces caractéristiques des modèles démontrent la possibilité de prédire avec succès le DMB à partir de données de télédétection.

Mots clés : diamètre de la branche, modèle, inventaire forestier

Introduction
There is growing recognition that forest sector competitiveness can be increased by capitalizing on the superior attributes of wood fibre (Barbour and Kellogg 1990) and by optimizing the use of wood fibre, ideally all along the value chain spanning forest to market (Mackenzie and Bruemmer 2009). Such optimization requires information about wood quality as well as about timber volume. Forest resource inventories in many jurisdictions typically provide information about stand composition, age, height and site quality, but not about wood quality. Consequently, information about wood quality must be added to forest resource inventories to support value chain optimization.

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Knots strongly affect the value of lumber through their influence on the strength and appearance of sawn wood. For example, an increase in maximum edge knot diameter from 1.9 cm to 3.2 cm lowers the grade of North American nominal 2” x 4” lumber by two classes from Select Structural to No. 2 (NLGA 2003, Benjamin et al. 2007). Since knots arise from tree branches, the size of knots is closely related to branch diameter. Estimates of maximum branch diameter (MBD) would provide a measure of the maximum knot size in the crown region of the stem, and an indication of maximum knot size below this region.

It is not feasible to directly measure MBD for inclusion in forest resource inventories, but a promising approach is to develop relationships between MBD and features such as tree height, crown radius and measures of competitive status. These features are often not available in forest inventories, but advances in remote sensing are making it feasible to successfully estimate individual tree features using automated approaches. In particular, LiDAR can be used to resolve features of individual tree crowns (Wulder et al. 2008). Tree height can be estimated from LiDAR densities as low as 1 return per m², but estimation of features such as crown diameter requires higher densities or combined analysis with digital imagery (Leckie et al. 2003, Wulder et al. 2008).

A number of models have been developed to estimate the MBD or branch basal area within whorls. Branch diameter increases with distance below the tree apex, although for all but trees in young stands, the greatest diameter typically occurs above the crown base (Mäkinen et al. 2003, Garber and Maguire 2005, Weiskittel et al. 2010). Whorl level models also indicate that maximum branch diameter increases with tree characteristics representative of tree size and growing space, such as diameter at breast height (DBH) crown width, tree height, and tree spacing (Garber and Maguire 2005, Hein et al. 2007, Benjamin et al. 2009, Weiskittel et al. 2010).

Crown width or radius is a logical starting point in the development of models to estimate MBD from remotely sensed data. Not only is it becoming feasible to obtain measurements of crown radius through remote sensing techniques, but a strong allometric relationship exists between branch diameter and branch length or crown width (Cannell et al. 1988, Garber and Maguire 2005, Fernández and Norero 2006). This relationship satisfies the structural requirements of a branch acting as a flexible beam attached to the tree stem (Bertram 1989, Castéra and Morlier 1991).

Maximum branch diameter may be reduced in trees with lower social status or reduced competitive status. Decreasing maximum branch diameter has been observed with increasing tree slenderness coefficient (tree height/DBH) (Garber and Maguire 2005, Hein et al. 2007), a variable that is reflective of tree social position. Garber and Maguire’s (2005) branch diameter model also predicted lower branch diameter with decreasing relative tree height (tree height/height of tallest tree).

The objectives of this study are to: (i) develop models of maximum diameter of live branches using independent variables that characterize crown size, tree size, and competitive status and that also could be obtained through remote sensing; and (ii) determine whether silvicultural treatments affect MBD relationships. This study examines MBD relationships for six conifer and three broadleaved species, with data obtained mainly from silvicultural experiments across Canada. The variation in crown radius resulting from the silvicultural treatments provides an effective basis for developing models, which all incorporated this variable. Although the intended application for this work is to estimate MBD using remotely sensed data, ground-based field data were used in this study. Because measurement of individual crown characteristics by remote sensing is an emerging technology, remotely sensed crown data were not available for the study sites. Instead, ground-based measurements were used for model development.

Materials and Methods

Study sites

Maximum diameter of live branches data were obtained for economically important conifer species from across Canada: balsam fir (Abies balsamea [L.] Mill.), white spruce (Picea glauca [Moench] Voss), black spruce (Picea mariana [Mill.] BSP), jack pine (Pinus banksiana Lamb.), lodgepole pine (Pinus contorta Dougl.), and Douglas-fir (Pseudotsuga menziesii [Mirb.] Franco). Additionally, a smaller amount of data was obtained for three broadleaved species in Quebec: yellow birch (Betula alleghaniensis Britt.), white birch (Betula papyrifera Marsh.), trembling aspen (Populus tremuloides Michx.). Many of the data were obtained from density management (initial spacing or thinning) experiments (Table 1), which provided a wide range of crown characteristics, and allowed examination of silvicultural treatment effects on MBD relationships at two locations.

Branch and tree measurement

The diameter of the largest living branches on trees was measured to the nearest 0.1 cm using diameter tapes or callipers. Larger trees were felled or climbed to access the branches, whereas it was possible to measure branches on shorter trees from the ground. The height of standing trees was measured by hypsometer, whereas the height of felled trees was measured by tape. The definition of crown base varied somewhat among data sources, but can be generalized as the height at which foli a becomes continuous. Height to crown base was measured by hypsometer or height pole on standing trees and by tape on felled trees. For some data sources, crown radius was measured in the four cardinal directions by sighting vertically to determine the crown edge and then measuring the distance to the centre of the stem. For other data sources, crown width was measured in the N–S and E–W directions using a handheld laser measurement device (LaserAce, Aberdeen, Scotland). For all data sources, the mean crown radius was calculated as one-half of the geometric average of N–S and E–W crown widths.

MBD models

Power functions were used to relate the mean diameter of the three largest branches per stem to crown radius, tree height and a variable used as a surrogate for competition. The three variables used as proxies for competition were crown length (eq. 1a), stand basal area (eq. 1b) and basal area of the stems larger than the tree of interest (Eq. 1c). Eq. 1a requires only individual-tree independent variables, whereas eq. 1b and eq. 1c additionally require stand-level independent variables. In eq. 1c, one unit was added to the competition index in order to include the largest trees of the stand (i.e., when $BAL_{sp} = 0$ m²/ha) in the analysis.
Table 1. Characteristics of study sites for maximum branch diameter (MBD) data

<table>
<thead>
<tr>
<th>Species</th>
<th>Study Location</th>
<th>Treatment/condition</th>
<th>Lat. (°N), Long. (°W) (mean for studies with multiple sites)</th>
<th>Stand age or time since previous harvest</th>
<th>Dominant height (m)</th>
<th>MBD (cm)</th>
<th>Crown radius (m)</th>
<th>No. of trees sampled</th>
<th>Reference for site and treatment description</th>
</tr>
</thead>
<tbody>
<tr>
<td>balsam fir</td>
<td>Green River, NB</td>
<td>PCT</td>
<td>47°46', 68°15'</td>
<td>53–62</td>
<td>21.3 to 22.0</td>
<td>2.1 to 2.7</td>
<td>1.0 to 1.5</td>
<td>161</td>
<td>Pitt and Lanteigne (2008)</td>
</tr>
<tr>
<td>balsam fir</td>
<td>Gaspé, QC</td>
<td>N</td>
<td>48°39', 66°02'</td>
<td>–</td>
<td>21.3 to 23.8</td>
<td>3.3</td>
<td>2.1</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>white spruce</td>
<td>Petawawa, ON</td>
<td>IS</td>
<td>45°57', 77°27'</td>
<td>42</td>
<td>16.6 to 17.9</td>
<td>2.1 to 5.2</td>
<td>1.1 to 1.3 57</td>
<td>–</td>
<td>Petrinovic et al. (2009)</td>
</tr>
<tr>
<td>white spruce</td>
<td>several sites, QC</td>
<td>P</td>
<td>48°04', 69°00'</td>
<td>19</td>
<td>Approx 5.8 to 7.2</td>
<td>2.0</td>
<td>1.4</td>
<td>155</td>
<td></td>
</tr>
<tr>
<td>white spruce</td>
<td>Calling Lake, AB</td>
<td>N</td>
<td>55°06', 113°02'</td>
<td>150</td>
<td>32.2</td>
<td>2.6</td>
<td>2.8</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>white spruce</td>
<td>Gaspé, QC</td>
<td>N</td>
<td>48°41', 66°58'</td>
<td>–</td>
<td>23.8 to 24.0</td>
<td>4.4</td>
<td>2.2</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>black spruce</td>
<td>Kapuskasing, ON</td>
<td>IS</td>
<td>49°01', 82°09'</td>
<td>25</td>
<td>7.1 to 8.9</td>
<td>1.0 to 2.0</td>
<td>0.7 to 1.2</td>
<td>352</td>
<td>Groot (2002)</td>
</tr>
<tr>
<td>black spruce</td>
<td>Cochrane, ON</td>
<td>UE</td>
<td>49°07', 80°36'</td>
<td>78</td>
<td>12.2 to 16.8</td>
<td>1.8 to 2.1</td>
<td>0.8 to 1.1</td>
<td>32</td>
<td></td>
</tr>
<tr>
<td>black spruce</td>
<td>Gaspé, QC</td>
<td>N</td>
<td>48°35', 66°10'</td>
<td>–</td>
<td>17.0 to 20.3</td>
<td>2.7</td>
<td>1.3</td>
<td>14</td>
<td></td>
</tr>
<tr>
<td>Douglas-fir</td>
<td>Shawnigan Lake, BC</td>
<td>CT</td>
<td>48°38', 123°43'</td>
<td>60</td>
<td>25.2 to 27.1</td>
<td>2.4 to 4.4</td>
<td>1.2 to 3.1</td>
<td>28</td>
<td></td>
</tr>
<tr>
<td>jack pine</td>
<td>Petawawa, ON</td>
<td>IS</td>
<td>45°59', 77°25'</td>
<td>36</td>
<td>20.8</td>
<td>3.1 to 4.2</td>
<td>1.5 to 2.2</td>
<td>16</td>
<td></td>
</tr>
<tr>
<td>jack pine</td>
<td>Smurfit-Stone, QC</td>
<td>P and N</td>
<td>48°22', 74°00'</td>
<td>21 to 79</td>
<td>5.7 to 19.6</td>
<td>2.2 to 3.0</td>
<td>1.2 to 1.3</td>
<td>50</td>
<td></td>
</tr>
<tr>
<td>jack pine</td>
<td>Eel River</td>
<td>PCT</td>
<td>46°59', 65°01'</td>
<td>56</td>
<td>17.2</td>
<td>2.3 to 3.1</td>
<td>1.0 to 1.4</td>
<td>18</td>
<td></td>
</tr>
<tr>
<td>jack pine</td>
<td>Timmins, ON</td>
<td>N</td>
<td>48°22', 81°26'</td>
<td>50–90</td>
<td>22.1 to 24.9</td>
<td>2.9 to 3.1</td>
<td>1.1</td>
<td>142</td>
<td></td>
</tr>
<tr>
<td>lodgepole pine</td>
<td>Teepee Pole South, AB</td>
<td>IS and N</td>
<td>51°54', 115°12'</td>
<td>67</td>
<td>17.5 to 18.8</td>
<td>1.9 to 3.8</td>
<td>0.8 to 1.5</td>
<td>9</td>
<td></td>
</tr>
<tr>
<td>yellow birch</td>
<td>Gaspé, QC</td>
<td>N</td>
<td>48°39', 67°04'</td>
<td>–</td>
<td>21.0 to 24.0</td>
<td>14.0</td>
<td>4.0</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>white birch</td>
<td>Gaspé, QC</td>
<td>N</td>
<td>48°37', 67°13'</td>
<td>–</td>
<td>15.7 to 19.3</td>
<td>5.8</td>
<td>1.8</td>
<td>9</td>
<td></td>
</tr>
<tr>
<td>trembling aspen</td>
<td>Petawawa, ON</td>
<td>N</td>
<td>48°39', 67°14'</td>
<td>–</td>
<td>17.9 to 19.3</td>
<td>7.3</td>
<td>2.7</td>
<td>7</td>
<td></td>
</tr>
</tbody>
</table>

1PCT = precommercial thinning; CT = commercial thinning; IS = initial spacing; UE = uneven-aged management; N = naturally regenerated stand; P = planted stand
Note: Maximum branch diameters and crown radius values are arithmetic means for treatments or condition.

\[[1a]\]
\[
MBD_{ijk} = b_1 \cdot R_{ijk}^b \cdot H_{ijk}^h \cdot L_{ijk}^l + \epsilon_{ijk}
\]

\[[1b]\]
\[
MBD_{ijk} = b_1 \cdot R_{ijk}^b \cdot H_{ijk}^h \cdot BA_{ijk}^a + \epsilon_{ijk}
\]

\[[1c]\]
\[
MBD_{ijk} = b_1 \cdot R_{ijk}^b \cdot H_{ijk}^h \cdot BA_{ijk}^a \cdot (1 + BAL_{ijk})^h + \epsilon_{ijk}
\]

where \( b_i \) to \( b_s \) are estimated fixed effect parameters; \( ijk \) are indices that refer to the hierarchical level of the information, \( i \) represents location, \( j \) the plot within location \( i \) and \( k \) the tree that is in plot \( j \); \( MBD_{ijk} \) is the mean branch diameter of the three largest branches (cm); \( R_{ijk} \) is the mean crown radius (m); \( H_{ijk} \) is the total stem height (m); \( L_{ijk} \) is the crown length (m); \( BA_{ijk} \) is the plot basal area (m²/ha); \( BAL_{ijk} \) is the competition index, defined as the basal area of the stems larger than that of stem \( k \) (m²/ha).

For models 1a and 1b, random effects were included in the slope parameter \((b_1)\) such that:

\[[2]\]
\[
b_1 = b_{1.base} + b_{1.species}
\]

where \( b_{1.base} \) is a location random effect, with \( b_{1.base} \sim N(0, \sigma_{b_{1.base}}^2) \).

Plot random effects in the three models and the location random effect in eq. 1c were found to be non-significant. Moreover, heteroscedasticity was modeled using an exponential function for eq. 1b and eq. 1c (Pinheiro and Bates 2000):

\[[3]\]
\[
\epsilon_{ijk} \sim N\left(0, \sigma_{res}^2 \cdot \exp^{\gamma X_{ijk}}\right)
\]

where \( \sigma_{res}^2 \) is the residual variance and \( \gamma \) is the estimated variance parameter.

For eq. 1a, no variance function was found to increase the quality of the fit:

\[[4]\]
\[
\epsilon_{ijk} \sim N(0, \sigma_{res}^2)
\]

The models were calibrated using the gnl (eq. 1c) or nlme (eq. 1a, eq. 1b) procedures in R (R Development Core Team 2008).

The effect of species on each parameter was also tested. This was done by setting a base value for balsam fir, and testing if the change between balsam fir and each species was significantly different than zero (eq. 5):

\[[5]\]
\[
b_{x} = b_{x.base} + b_{x.species}
\]

where \( x \) is the subscript to indicate parameter number in eq. 1a to eq. 1c; \( b_{x.base} \) is the base value of the parameter, i.e., the group to which each species parameter is compared, and \( b_{x.species} \) is the change in parameter for a specific species.

The species were dropped from the model in the same way Littell et al. (1996) suggest for linear mixed effect models: the least significant species parameter was eliminated, by group-
ing the dropped species into the base value category, and recalibrating the model. This was repeated until all of the parameters in the model were found to be significant (P < 0.05). Moreover, the variance was assumed to be different for each species, by estimating species-specific parameters of the variance function (eq. 6a and eq. 6b).

\[ 6a \] \[ \hat{e}_{ik} \sim N(0, \sigma_{re}^2 \cdot \gamma l ) \] for eq. 1b and eq. 1c

\[ 6b \] \[ \hat{e}_{ik} \sim N(0, \sigma_{re}^2 \cdot \gamma l ) \] for eq. 1a

where \( l \) is the subscript to identify species grouping level and \( \gamma \) is the species-specific variance function parameter.

The question of whether patterns of model residuals were related to silvicultural treatment was examined with the Green River balsam fir data, which comprised three replications of four precommercial thinning treatments, and the Stringer black spruce data, which comprised two replications of seven initial spacing treatments. One-way ANOVA (Green River) and linear regression (Stringer) were used to test whether bias was related to silvicultural treatment.

Because usual approaches to model validation are of little actual benefit in evaluating models (Kozak and Kozak 2003, Yang et al. 2004), the model fit to a split portion of the data set was not examined. Instead the entire data set was used for model fitting.

**Results**

In the models where species was not considered, mean diameter of the largest branches was proportional to crown radius (positive \( b_2 \)), crown length (positive \( b_4 \) for eq. 1a) and stem height (positive \( b_3 \)), and was inversely proportional to stand basal area (negative \( b_7 \)) and basal area of the stems larger than that of the sample stem (negative \( b_2 \) ) (Table 2). Moreover, the general quality of the fit \( (R^2, \text{RMSE}) \) increased as stand and competition factors are included in the model. The Akaike information criterion \( (\text{AIC}) \) cannot be used to compare the different models since their estimation were not carried out with the same data. The AIC can, however, be used to compare the effect of including species-specific parameters within the model on the quality of the fit.

The models were greatly enhanced when species-specific parameters were used (Tables 2 and 4). The most important changes were for models 1a and 1c, where the AIC was halved when species was included in both the variance and fixed effect parameters. For the models with stand basal area (eq. 1b), the AIC was also reduced, but to a lesser extent. Model 1c with species included showed no significant bias with respect to crown radius, stand basal area, basal area of larger trees, or DBH (Fig. 1).

In the species-specific models, all of the species presented the same trends as those observed in the models that did not resolve species. The trends varied among species, however, with the broadleaf species (trembling aspen, yellow and white birch) presenting generally larger maximum branch diameters than the coniferous species (balsam fir, Douglas-fir, lodgepole pine, jack pine, white and black spruce) (Fig. 2). The individual-tree-based model (eq. 1a) predicted that white birch and trembling aspen have similar-sized branches, whereas yellow birch will have the largest branches for a given set of dendrometric variables. Moreover, the prediction errors for the deciduous species were larger than those of the coniferous species, as can be seen by the values of \( \gamma l \) (Table 3). Black spruce had the steepest increment with respect to crown radius of all the species (greatest \( b_2 \) values), although the range of crown radius in the fitting data was less for this species than other species (Table 1). The individual-tree-based model (eq. 1a) predicted similar-sized mean branch diameters for the other coniferous species (balsam fir, Douglas-fir, lodgepole pine, jack pine, and white spruce). Lastly, balsam fir, Douglas-fir and lodgepole pine were not discriminated by the model, with the three species having the same values for each parameter. The differences between these species occurred in the variance function parameter, where the error of the prediction increased from balsam fir \( (\gamma l =1.0415) \) to Douglas-fir \( (\gamma l =1.2869) \) to Lodgepole pine \( (\gamma l =2.2799) \) (Table 3).

When crown length was substituted by stand basal area as a measure of competitive status in the model (eq. 1b), the differences between the different coniferous species was slightly more apparent (Fig. 3). The model was not able to separate balsam fir and Douglas-fir (Table 3). Moreover, the model predicted that the mean branch diameter of the three largest branches of jack pine and white spruce are very similar. The difference occurred in the variance parameter (Table 3), which is lower for white spruce.

### Table 2. Parameter estimates (standard deviations in parentheses) and fit statistics for the general models

<table>
<thead>
<tr>
<th>Parameter</th>
<th>eq. 1a</th>
<th>eq. 1b</th>
<th>eq. 1c</th>
</tr>
</thead>
<tbody>
<tr>
<td>( b_1 )</td>
<td>1.1330 (0.1531)</td>
<td>0.7823 (0.0847)</td>
<td>0.8399 (0.0356)</td>
</tr>
<tr>
<td>( b_2 )</td>
<td>0.9031 (0.0267)</td>
<td>0.5420 (0.0228)</td>
<td>0.4760 (0.0257)</td>
</tr>
<tr>
<td>( b_3 )</td>
<td>0.1529 (0.0481)</td>
<td>0.6084 (0.0382)</td>
<td>0.6340 (0.0265)</td>
</tr>
<tr>
<td>( b_4 )</td>
<td>0.0685 (0.0298)</td>
<td>-0.1821 (0.0201)</td>
<td>-0.1582 (0.0238)</td>
</tr>
<tr>
<td>( b_5 )</td>
<td>-</td>
<td>-0.0544 (0.0112)</td>
<td>-</td>
</tr>
</tbody>
</table>

**Variance–covariance parameters**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>eq. 1a</th>
<th>eq. 1b</th>
<th>eq. 1c</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma_{re}^2 )</td>
<td>0.3068</td>
<td>0.1667</td>
<td>0.1525</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>-</td>
<td>0.5360</td>
<td>0.6922</td>
</tr>
<tr>
<td>( \sigma_l )</td>
<td>0.6609</td>
<td>0.0992</td>
<td>-</td>
</tr>
</tbody>
</table>

**Fit statistics**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>eq. 1a</th>
<th>eq. 1b</th>
<th>eq. 1c</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pseudo-R2</td>
<td>0.58</td>
<td>0.78</td>
<td>0.83</td>
</tr>
<tr>
<td>Pseudo-RMSE</td>
<td>0.99</td>
<td>0.45</td>
<td>0.40</td>
</tr>
<tr>
<td>AIC</td>
<td>2230</td>
<td>375</td>
<td>462</td>
</tr>
</tbody>
</table>

\( P \) calculated without random effects as 1 – \[ \sum_{i=1}^{n} (\hat{y}_{ik} - \hat{y}_{ik})^2 \] \[ \sum_{i=1}^{n} (\hat{y}_{ik} - \bar{y})^2 \]

\( P \) calculated without random effects as \[ \left( \sum_{i=1}^{n} (\hat{y}_{ik} - \hat{y}_{ik})^2 \right) ^{0.5} \] where \( n \) is the number of measurements.
Table 3. Parameter estimates (standard deviations in parentheses) and fit statistics for models 1a, 1b and 1c with species-specific parameters

<table>
<thead>
<tr>
<th></th>
<th>(b_1)</th>
<th>(b_2)</th>
<th>(b_3)</th>
<th>(b_4)</th>
<th>(b_5)</th>
<th>(\gamma_1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>eq. 1a</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>base (balsam fir)</td>
<td>0.3323</td>
<td>0.4378</td>
<td>0.5406</td>
<td>0.1471</td>
<td>–</td>
<td>1.0415</td>
</tr>
<tr>
<td></td>
<td>(0.0667)</td>
<td>(0.0269)</td>
<td>(0.0564)</td>
<td>(0.0267)</td>
<td>–</td>
<td></td>
</tr>
<tr>
<td>black spruce</td>
<td>0.8639</td>
<td>0.3085</td>
<td>-0.4522</td>
<td>n.s.</td>
<td>–</td>
<td>0.9320</td>
</tr>
<tr>
<td></td>
<td>(0.1639)</td>
<td>(0.0450)</td>
<td>(0.0709)</td>
<td></td>
<td>–</td>
<td></td>
</tr>
<tr>
<td>Douglas-fir</td>
<td>n.s.</td>
<td>n.s.</td>
<td>n.s.</td>
<td>n.s.</td>
<td>–</td>
<td>1.2869</td>
</tr>
<tr>
<td>Jack pine</td>
<td>n.s.</td>
<td>0.1322</td>
<td>n.s.</td>
<td>n.s.</td>
<td>–</td>
<td>1.6758</td>
</tr>
<tr>
<td></td>
<td>(0.0436)</td>
<td></td>
<td></td>
<td></td>
<td>–</td>
<td></td>
</tr>
<tr>
<td>lodgepole pine</td>
<td>n.s.</td>
<td>n.s.</td>
<td>n.s.</td>
<td>n.s.</td>
<td>–</td>
<td>2.2799</td>
</tr>
<tr>
<td>white spruce</td>
<td>0.4466</td>
<td>n.s.</td>
<td>-0.4801</td>
<td>0.2727</td>
<td>–</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.1158)</td>
<td></td>
<td>(0.0732)</td>
<td>(0.0440)</td>
<td>–</td>
<td>1.0000</td>
</tr>
<tr>
<td>trembling aspen</td>
<td>0.3813</td>
<td>n.s.</td>
<td>n.s.</td>
<td>n.s.</td>
<td>–</td>
<td>3.7899</td>
</tr>
<tr>
<td></td>
<td>(0.0716)</td>
<td></td>
<td></td>
<td></td>
<td>–</td>
<td></td>
</tr>
<tr>
<td>white birch</td>
<td>n.s.</td>
<td>n.s.</td>
<td>0.2873</td>
<td>n.s.</td>
<td>–</td>
<td>6.7268</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.0488)</td>
<td></td>
<td>–</td>
<td></td>
</tr>
<tr>
<td>yellow birch</td>
<td>n.s.</td>
<td>n.s.</td>
<td>0.3643</td>
<td>n.s.</td>
<td>–</td>
<td>10.3656</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.0329)</td>
<td></td>
<td>–</td>
<td></td>
</tr>
<tr>
<td>eq. 1b</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>base (balsam fir)</td>
<td>0.4987</td>
<td>0.4834</td>
<td>0.5842</td>
<td>-0.0697</td>
<td>–</td>
<td>0.5244</td>
</tr>
<tr>
<td></td>
<td>(0.0752)</td>
<td>(0.0262)</td>
<td>(0.0546)</td>
<td>(0.0332)</td>
<td>–</td>
<td></td>
</tr>
<tr>
<td>black spruce</td>
<td>1.0022</td>
<td>0.2512</td>
<td>-0.3312</td>
<td>-0.1091</td>
<td>–</td>
<td>0.4849</td>
</tr>
<tr>
<td></td>
<td>(0.1847)</td>
<td>(0.0509)</td>
<td>(0.0818)</td>
<td>(0.0414)</td>
<td>–</td>
<td></td>
</tr>
<tr>
<td>Douglas-fir</td>
<td>n.s.</td>
<td>n.s.</td>
<td>n.s.</td>
<td>n.s.</td>
<td>–</td>
<td>0.4278</td>
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<tr>
<td>Jack pine</td>
<td>n.s.</td>
<td>n.s.</td>
<td>0.3200</td>
<td>-0.1847</td>
<td>–</td>
<td>0.6960</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.0864)</td>
<td>(0.0710)</td>
<td>–</td>
<td></td>
</tr>
<tr>
<td>white spruce</td>
<td>n.s.</td>
<td>n.s.</td>
<td>0.4773</td>
<td>-0.3006</td>
<td>–</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.1133)</td>
<td>(0.0857)</td>
<td>–</td>
<td>0.4882</td>
</tr>
<tr>
<td>eq. 1c</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>base (balsam fir)</td>
<td>0.7484</td>
<td>0.4216</td>
<td>0.5790</td>
<td>-0.1123</td>
<td>-0.0632</td>
<td>0.5391</td>
</tr>
<tr>
<td></td>
<td>(0.0861)</td>
<td>(0.0286)</td>
<td>(0.0410)</td>
<td>(0.0216)</td>
<td>(0.0104)</td>
<td></td>
</tr>
<tr>
<td>black spruce</td>
<td>0.9421</td>
<td>0.3043</td>
<td>-0.4100</td>
<td>n.s.</td>
<td>n.s.</td>
<td>0.5292</td>
</tr>
<tr>
<td></td>
<td>(0.1976)</td>
<td>(0.0487)</td>
<td>(0.0673)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Douglas-fir</td>
<td>n.s.</td>
<td>-0.0989</td>
<td>n.s.</td>
<td>n.s.</td>
<td>n.s.</td>
<td>0.4362</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0393)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jack pine</td>
<td>n.s.</td>
<td>n.s.</td>
<td>0.0725</td>
<td>n.s.</td>
<td>n.s.</td>
<td>0.8110</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.0084)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>white spruce</td>
<td>n.s.</td>
<td>0.1955</td>
<td>n.s.</td>
<td>n.s.</td>
<td>n.s.</td>
<td>0.4975</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0534)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: for \(b_1\) to \(b_5\), \(b_x = b_{x,\text{base}} + b_{x,\text{species}}\). For n.s. (non-significant) \(b_{x,\text{species}}\), the parameter for the species is equivalent to the base parameter.

Table 4. Variance–covariance parameters and fit statistics for models 1a, 1b and 1c with species-specific parameters

<table>
<thead>
<tr>
<th>Variance–covariance parameters</th>
<th>Fit statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\sigma_{\text{res}})</td>
<td>(\sigma_i)</td>
</tr>
<tr>
<td>eq. 1a</td>
<td>0.3118</td>
</tr>
<tr>
<td>eq. 1b</td>
<td>0.1594</td>
</tr>
<tr>
<td>eq. 1c</td>
<td>0.1512</td>
</tr>
</tbody>
</table>
As with model 1a, black spruce showed the greatest change with respect to crown radius. Differences between species became noticeable when the basal area of the larger trees was included in the model (eq. 1c). Balsam fir and Douglas-fir had different values for $b_2$ (Table 3), leading to differences in the MBD to crown radius curves (Fig. 4). Moreover, jack pine and white spruce also presented different trends with respect to crown radius and tree height, which was not the case for models 1a and 1b. Finally, the location random effect could not be estimated when the basal area of the larger stems is included in the model.

For the Green River balsam fir data, analysis of variance indicated that precommercial thinning treatments did not have a significant effect on model prediction error (p = 0.76, total df = 11, treatment df = 3). For the Stringer black spruce initial spacing data, linear regression analysis indicated that model prediction error was not significantly related to initial spacing (p = 0.35, total df = 12).

Discussion

The models developed in this study are well-suited for predicting MBD for inclusion in forest inventory. The independent variables can be estimated through emerging remote sensing technologies including LiDAR, high-resolution digital photography and algorithms for delineating individual tree crowns (Pitt and Pineau 2009). As suggested by Briggs et al. (2008), application of such models would allow mapping of mean MBD across mosaics of forest stands. The models developed in this study appear to be the first that would additionally allow characterization of the variability of MBD within stands, both within and between species, using individual tree input data that could be obtained solely through remote sensing.

The low RMSE of models [1b] and [1c] (about 3 mm) and lack of bias with respect to crown radius, stand basal area and basal area of larger trees, and DBH indicate that accurate predictions of individual tree MBD from crown and stand features are possible. Predictions of average MBD and the distribution of MBD for stands or for components of stands should have lower error in accordance with the central limit theorem. Briggs et al. (2008) were able to fit mean tree (i.e., stand-level) models with RMSE of about 2 mm. It should be emphasized, however, that the magnitude of error increases as MBD increases.

The strong relationship of MBD to crown radius evident in all models was expected, given the mechanical requirements of tree branch form (Bertram 1989, Castéra and Morlier 1991). Simply put, longer branches must be thicker in order to support themselves. The exponent of crown radius, $b_2$, was less than unity in all models, consistent with relationships between branch diameter and length observed in other studies (Burk et al. 1983, Deleuze et al. 1996, Fernández and Norero 2006). Bertram (1989) noted that an exponent of 2 is
necessary to maintain a constant stress in a cantilever beam, and an exponent of 1.5 is necessary to maintain a constant deflection at the free end of the branch, relative to length. An exponent less than one indicates that branches become more slender (greater length/diameter) as they become larger, and that the main constraint of branch architecture may be to avoid mechanical failure (Bertram 1989).

The increase in MBD with height (positive $b_3$ in all models) may reflect the exposure of taller trees to greater wind speeds. Increased branch diameter in response to wind load has been observed in *Pinus radiata* (Watt et al. 2005). Wind increases the deflection of branches not only directly, but also indirectly through motion transmitted by swaying of the tree stem. Tree branches must become thicker for a given length to counteract the additional mechanical load imposed by wind.

The decrease in MBD with basal area (negative $b_4$ in models 1b and 1c) and basal area of larger trees (negative $b_5$ in model 1c) is likely an effect of increasing competition on branch architecture. The decreased foliar mass carried by branches growing under lower light levels reduces the branch load-bearing requirement. Although it is feasible to estimate stand basal area and basal area of larger trees using remote sensing observations, a future refinement of MBD models would be to replace these variables with crown area and crown area of larger trees. Crown variables can be estimated more directly from remote sensing observations, simplifying the estimation of competitive effects on crown architecture.

A unique aspect of this study is that it compares MBD diameter relationships among species. The much thicker branches of trembling aspen and the birches compared with the conifers for the same crown radius, tree height or crown length indicates that crown architecture adaptations for excurrent versus decurrent growth forms are fundamentally different. Trees with excurrent growth forms (typically conifers, but also some broadleaved trees) are frequently observed to be less resistant to damage from snow, ice loading, and wind loading than trees with decurrent growth forms (Warrillow and Mou 1999, Yorks and Adams 2005, Duryea et al. 2007). The thinner, more flexible branches of the excurrent form can shed moderate loads, but are more likely to fail as loads increase.

---

**Fig. 2.** Predicted mean diameter of the three largest branches per species using the individual-tree species-specific model (eq. 1a) versus: (a) crown radius with constant tree height (mean value in data: 12.9 m) and crown length (mean value in the data: 6.5 m); (b) tree height with constant crown length (6.5 m) and crown radius (mean value in the data: 1.3 m); and, (c) crown length with constant tree height (12.9 m) and crown radius (1.3 m).
than the thicker branches of the decurrent form. Although there were also differences among the conifers, they were much smaller than the differences between conifers and broadleaves. This result is in agreement with the findings of Weiskittel et al. (2010), who concluded that while including species differences improved estimates in models of conifer maximum branch diameter, crown size variables accounted for a high proportion of the variation.

The absence of treatment-related bias in MBD estimates from Model [1c] for the Green River balsam fir precommercial thinning and Kapuskasing black spruce initial density experiments indicates that density management effects on northern conifer MBD are fully expressed through effects on crown radius, stand basal area and basal area of larger trees. This result is consistent with the findings by Grotta et al. (2004) and Briggs et al. (2008) that density, fertilization and species mixture treatments had no effect on Douglas-fir branch diameter after tree-level variables were taken into account. It is also consistent with the conclusion of Fernández and Norero (2006) that relationships between branch length and diameter are stable over sites and management conditions. Garber and Maguire (2005) also found that tree-level variables accounted for most of the variation in MBD, but treatment effects were still significant in their models.

The models presented in this paper predict the mean diameter of the three largest live branches of a tree, but not the diameter of dead branches. If the base of the living crown is above the first logs, then the models will not provide information about past branch diameter and knot size in the most valuable part of the tree. Previous studies have shown that the largest branches are found above the crown base (Mäkinen et al. 2003, Garber and Maguire 2005, Hein et al. 2007), suggesting that estimates of current maximum branch diameter provide an approximate upper limit to past maximum branch diameters. Observations of knot diameters along the stem of Scots pine (Moberg 1999) are broadly consistent with this supposition; knot sizes remained more or less constant between the base of the crown and the butt region. To better incorporate product quality into remote sensing-based resource inventory, the next step will be to establish relationships between living branch diameters and branch characteristics below the crown base, and more importantly between crown characteristics and knottiness in the wood products.

Although ground-based measurements of the models' independent variables were used in this study, the estimation of these variables by remote sensing is an emerging cost-effective, operational technology (Leckie et al. 2003, Wulder et al. 2008). As these remote sensing estimates become routine, the models developed in this study offer a feasible approach to the estimation of MBD for incorporation into forest resource inventories. The Canadian Wood Fibre Centre is also carrying out research to estimate other attributes influencing wood quality, (e.g., DBH, sapwood area, and wood density) from crown and stand variables, which will potentially broaden the suite of wood quality attributes that can be added to forest inventories.

In summary, the models presented in this paper show that it is possible to accurately (RMSE of about 3 mm) predict mean branch diameter of the largest branches within the

Fig. 3. Predicted mean diameter of the three largest branches per species using the model with stand basal area (eq. 1b) versus: (a) crown radius with constant tree height (12.9 m) and stand basal area (mean value in the data: 24 m²/ha); (b) tree height with constant crown length (6.5 m) and stand basal area (24 m²/ha); and (c) versus stand basal area with constant tree height (12.9 m) and crown radius (1.3 m).
crown from information readily available from remote sensing information. Not only can these models provide useful information to forest managers wanting to evaluate wood quality of remotely sensed stands, it could also eventually be used as input into growth models that require more precise information on crown and branch architecture.

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References


